Signal subspace in time domain

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Summary

The notion of the signal and noise subspaces has been considered useful in biomagnetic signal processing. Signal Space Projection (SSP) uses it for artifact reduction [1], and the MUSIC algorithm uses it for source localization. So far, the signal and noise subspaces are defined in the spatial domain in which the signal subspace is defined as the span of the source lead field vectors.

This paper proposes for the first time (as far as the authors know) to define the signal subspace in the time domain. That is, the signal subspace is defined as the span of row vectors that contain the source time courses. (Such row vectors are referred to as the source time course vectors.)

By defining the time domain signal subspace in this manner, we can derive symmetric relationships between the time domain signal subspace and the spatial domain signal subspace. For example, while the sensor array outputs at a particular time point is expressed as a linear combination of the source lead field vectors, the outputs of a particular sensor is expressed as a linear combination of the source time course vectors.

Using the time-domain signal subspace, it is possible to interpret various interference removal methods that have been considered different as the time domain SSP. Such methods include the adaptive noise cancelling [2], sensor noise suppression [3], common temporal subspace projection [4], spatio-temporal tSSS[5] and recently proposed dual signal subspace projection [6]. Therefore, the notion of time-domain signal subspace can provide a broader perspective and useful insights over existing and new artifact/interference removal methods.

Reference

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[3] De Cheveigné, A., Simon, J.Z, *Journal of neuroscience methods*, *168* (2008):195-202.

[4] Tatanabe et al., 35th EMBC Annual International Conference of the IEEE, 3391-3394, 2013.

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Signal subspace in spatial domain

Sensor data : $\boldsymbol{y}(t) = \boldsymbol{y}_S(t) + \boldsymbol{\varepsilon}$

$$\underbrace{\boldsymbol{y}_S}_{\textbf{signal}} \qquad \underbrace{\boldsymbol{\varepsilon}}_{\textbf{noise}}$$

We assume that a total of Q discrete sources exist. Then, the signal vector is expressed as a linear combination of source lead field vectors.

$$\boldsymbol{y}_S(t) = \sum_{q=1}^{Q} s_q(t)\boldsymbol{l}_q$$

$\boldsymbol{l}_q$: lead field vector of the $q\mathrm{th}$ source

$s_q(t)$: activity of the $q\mathrm{th}$ source at time t

Thus, defining $E_S = span\{l_1, \ldots, l_Q\}$, then $y_S \in E_S$ holds

This E_S is referred to as the signal subspace

Signal subspace in time-domain

Data is measured at $t_1, \ldots, t_K$

The time-course (row) vector of the qth source is defined as

$$\boldsymbol{s}_q = [s_q(t_1), \ldots, s_q(t_K)]$$

$K_S = span\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_Q\}$ is defined as the signal subspace in time domain.

Data matrix $\boldsymbol{B}_S = \left[\boldsymbol{y}_S(t_1), \ldots, \boldsymbol{y}_S(t_K) \right]$

Symmetric relationships

Each column of $\boldsymbol{B}_S \in E_S$

Each row of $\boldsymbol{B}_S \in K_S$

Column span of $\boldsymbol{B}_S = E_S$

Row span of $\boldsymbol{B}_S = K_S$

Signal space projection (SSP) for interference suppression[1]

Data model: $\boldsymbol{B} = \boldsymbol{B}_S + \boldsymbol{B}_I + \boldsymbol{B}_\varepsilon$

Spatial domain SSP

- Assume the existence of interference-only data: $\boldsymbol{B}_e = \boldsymbol{B}_I + \boldsymbol{B}_\varepsilon$
- Using $\boldsymbol{B}_e$, the basis vectors of the interference subspace is obtained and the projector $\boldsymbol{P}_I$ is computed using the basis vectors.
- Interference suppression is attained by computing: $\left(\boldsymbol{I} - \boldsymbol{P}_I\right)\boldsymbol{B}$

Time domain SSP

If the projector to the time-domain interference subspace $\boldsymbol{\Pi}_I$ is obtained, interference suppression is attained by computing $\boldsymbol{B}\left(\boldsymbol{I} - \boldsymbol{\Pi}_I\right)$.

Various methods for denoising and interference suppression can be interpreted as time domain SSP. They differ only in a way how to derive $\boldsymbol{\Pi}_I$.

Adaptive noise cancelling (ANC)[2]

Data model

Sensor data :	$\boldsymbol{y}(t) = \boldsymbol{y}_S(t) + \boldsymbol{y}_I(t) + \boldsymbol{\varepsilon}$
Reference sensor data :	$\boldsymbol{x}(t) = \tilde{\boldsymbol{y}}_I(t) + \tilde{\boldsymbol{\varepsilon}}$

Regress $\boldsymbol{y}(t)$ with $\boldsymbol{x}(t)$: $\boldsymbol{y}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{v}(t)$

The residual term $\boldsymbol{v}(t)$ is the interference-removal results, and expressed as

$$\boldsymbol{v}(t) = \boldsymbol{y}(t) - \boldsymbol{\Sigma}_{yx}\boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{x}(t) \quad \text{where} \quad \boldsymbol{\Sigma}_{yx} = \left\langle \boldsymbol{y}\boldsymbol{x}^T \right\rangle, \quad \boldsymbol{\Sigma}_{xx} = \left\langle \boldsymbol{x}\boldsymbol{x}^T \right\rangle$$

Using data matrices, the residual term $\boldsymbol{V}$ is expressed as

$$\boldsymbol{V} = \boldsymbol{Y} - \boldsymbol{Y}\boldsymbol{X}^T(\boldsymbol{X}\boldsymbol{X}^T)^{-1}\boldsymbol{X} = \boldsymbol{Y}(\boldsymbol{I} - \boldsymbol{X}^T(\boldsymbol{X}\boldsymbol{X}^T)^{-1}\boldsymbol{X})$$

$\boldsymbol{X}^T(\boldsymbol{X}\boldsymbol{X}^T)^{-1}\boldsymbol{X}$ is the projector onto the row span of $\boldsymbol{X}$, and because the row span of $\boldsymbol{X}$ approximates the interference subspace, ANC can be interpreted as time-domain SSP.

Common temporal mode subspace projection (CtSP)[4]

Data model

Sensor data :	$\boldsymbol{B} = \boldsymbol{B}_S + \boldsymbol{B}_I + \boldsymbol{B}_\varepsilon$
Reference sensor data :	$\tilde{\boldsymbol{B}} = \tilde{\boldsymbol{B}}_I + \tilde{\boldsymbol{B}}_\varepsilon$

Considering

$$\mathrm{rspan}(\boldsymbol{B}) = \mathrm{rspan}(\boldsymbol{B}_S) \bigcup \mathrm{rspan}(\boldsymbol{B}_I) \bigcup \mathrm{rspan}(\boldsymbol{B}_\varepsilon)$$

$$\mathrm{rspan}(\tilde{\boldsymbol{B}}) = \mathrm{rspan}(\tilde{\boldsymbol{B}}_I) \bigcup \mathrm{rspan}(\tilde{\boldsymbol{B}}_\varepsilon)$$

We have

$$\begin{aligned}\mathrm{rspan}(\boldsymbol{B}) \bigcap \mathrm{rspan}(\tilde{\boldsymbol{B}}) &= \mathrm{rspan}(\boldsymbol{B}_I) \bigcup \left[\mathrm{rspan}(\boldsymbol{B}_\varepsilon) \bigcap \mathrm{rspan}(\tilde{\boldsymbol{B}}_\varepsilon)\right] \\ &= \mathrm{rspan}(\boldsymbol{B}_I) = K_I\end{aligned}$$

Deriving basis vectors of the intersection[1] $\mathrm{rspan}(\boldsymbol{B}) \bigcap \mathrm{rspan}(\tilde{\boldsymbol{B}})$,
we can get the projector onto K_{I} and the time-domain SSP is performed:

$$\hat{\boldsymbol{B}}_S = \boldsymbol{B}(\boldsymbol{I} - \boldsymbol{\Pi}_I)$$

The notation $\mathrm{rspan}(\boldsymbol{A})$ indicates the row span of the matrix $\boldsymbol{A}$.

[1]Golub and Van Loan (1996) Matrix computations. The Johns Hopkins University Press

Spatio-temporal signal space separation (tSSS)[5]

Data model (The method does not require reference sensor data.)

Sensor data : $\boldsymbol{B} = \boldsymbol{B}_S + \boldsymbol{B}_I + \boldsymbol{B}_\varepsilon$

- The sensor data is projected onto the internal and external regions of the sensor array using the SSS basis vectors.
- This separation can be done by using the SSS separators $\boldsymbol{\Gamma}_\mathrm{S}$ derived such that: $\boldsymbol{\Gamma}_S = \boldsymbol{C}\boldsymbol{C}^T(\boldsymbol{C}\boldsymbol{C}^T + \boldsymbol{D}\boldsymbol{D}^T)^{-1}$ where $\boldsymbol{C}$ and $\boldsymbol{D}$ are matrices whose columns are SSS basis vectors of the internal and external regions.

We can show:

$$\mathrm{rspan}(\boldsymbol{\Gamma}_S\boldsymbol{B}) \cap \mathrm{rspan}((\boldsymbol{I} - \boldsymbol{\Gamma}_S)\boldsymbol{B}) = \mathrm{rspan}(\boldsymbol{B}_I) = K_I$$

Deriving basis vectors of the intersection $\mathrm{rspan}(\boldsymbol{\Gamma}_S\boldsymbol{B}) \cap \mathrm{rspan}((\boldsymbol{I} - \boldsymbol{\Gamma}_S)\boldsymbol{B})$ we can get the projector onto K_I and the time-domain SSP is performed:

$$\hat{\boldsymbol{B}}_S = \boldsymbol{B}(\boldsymbol{I} - \boldsymbol{\Pi}_I)$$

Dual signal subspace projection (DSSP)[6]

Data model (The method does not require reference sensor data.)

Sensor data : $\boldsymbol{B} = \boldsymbol{B}_S + \boldsymbol{B}_I + \boldsymbol{B}_\varepsilon$

- The sensor data is projected onto the inside and outside the pseudo signal subspace.
- The projector $\boldsymbol{P}_\mathrm{S}$ is derived based on the span of sensor lead field over the source space.

Considering

$$\mathrm{rspan}(\boldsymbol{P}_S\boldsymbol{B}) = \mathrm{rspan}(\boldsymbol{P}_S\boldsymbol{B}_S) \bigcup \mathrm{rspan}(\boldsymbol{P}_S\boldsymbol{B}_I) \bigcup \mathrm{rspan}(\boldsymbol{P}_S\boldsymbol{B}_\varepsilon)$$

$$\mathrm{rspan}((\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B}) = \mathrm{rspan}((\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B}_I) \bigcup \mathrm{rspan}((\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B}_\varepsilon)$$

We have

$$\mathrm{rspan}(\boldsymbol{P}_S\boldsymbol{B}) \bigcap \mathrm{rspan}((\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B}) = \mathrm{rspan}(\boldsymbol{B}_I) = K_I$$

Deriving basis vectors of the intersection $\mathrm{rspan}(\boldsymbol{P}_S\boldsymbol{B}) \bigcap \mathrm{rspan}((\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B})$ we can get the projector onto K_I and the time-domain SSP is performed:

$$\hat{\boldsymbol{B}}_S = \boldsymbol{B}(\boldsymbol{I} - \boldsymbol{\Pi}_I)$$

Sensor noise suppression (SNS)[3]

Data model

Sensor data : $\boldsymbol{B} = \boldsymbol{B}_S + \boldsymbol{B}_\varepsilon$

Defining the j th row of $\boldsymbol{B}$ as $\boldsymbol{\beta}_j$, the assumption of the method is

$$\boldsymbol{\beta}_j^T = \mathrm{rspan}([\boldsymbol{\beta}_1^T, \ldots, \boldsymbol{\beta}_{j-1}^T, \boldsymbol{\beta}_{j+1}^T, \ldots, \boldsymbol{\beta}_M^T]^T)$$

Defining $\boldsymbol{\Omega}_j = [\beta_1^T, \ldots, \beta_{j-1}^T, \beta_{j+1}^T, \ldots, \beta_M^T]$, the j th row of $\boldsymbol{B}$

is expressed as $\boldsymbol{\beta}_j^T = \sum_{i \neq j} w_i \boldsymbol{\beta}_j^T = \boldsymbol{\Omega}_j \boldsymbol{W}_j$

Thus, denoised $\boldsymbol{\beta}_j$ is derived as

$$\hat{\boldsymbol{\beta}}_j = \boldsymbol{\beta}_j \boldsymbol{\Omega}_j (\boldsymbol{\Omega}_j^T \boldsymbol{\Omega}_j)^{-1} \boldsymbol{\Omega}_j^T$$

$\boldsymbol{\Omega}_j (\boldsymbol{\Omega}_j^T \boldsymbol{\Omega}_j)^{-1} \boldsymbol{\Omega}_j^T$ approximates the projector onto the row span of $\boldsymbol{B}$ and the row span of $\boldsymbol{B}$ approximates the row span of $\boldsymbol{B}_\mathrm{S}$, the signal subspace. Thus, SNS can be interpreted as time-domain SSP.

One example of cases in which the time-domain SSP interpretation is useful was simulated and the results are shown in the following slides.

- The CTF sensor array with six imaginary reference sensors (Fig.1) was assumed for data generation.
- A single source was assumed to exist 7-cm below the center of the array, and two interference sources having independent random time courses were assumed to exist 500-1000 cm far from the sensor array.
- Signal sensor time courses and interference-overlapped sensor time courses are shown in Fig. 2
- Reference sensor time courses and the results of ANC interference removal are shown in Fig. 3
- Next, low-frequency disturbance was added only to the reference sensor time courses. Results of ANC, as well as the reference sensor time courses, are shown in Fig. 4. *Surprisingly, there is no influence of the disturbance in the interference-removed results.*

- Interference data were re-generated with increasing the number of interference sources (from two) to six. The low-frequency disturbance was again added only to the reference sensor time courses. The results of ANC, as well as the reference sensor time courses, are shown in Fig. 6. The ANC method fails to remove the interference in this case.

- When the low-frequency disturbance was not added, the ANC can remove the interference. The reference sensor time courses and the results of ANC were shown in Fig. 7.

Interpretation of these results

When two interference sources exist, the relationship holds:

$$\mathrm{rspan}(\boldsymbol{X}) = K_I \cup \{\boldsymbol{d}\}$$

where $\{\boldsymbol{d}\}$ indicates the subspace spanned by the disturbance time course.

The orthogonal compliment of $K_I \cup \{\boldsymbol{d}\}$, which is the null space of the row span of $\boldsymbol{X}$, is approximately equal to the (time-domain) signal subspace, and the inclusion of $\{\boldsymbol{d}\}$ hardly affects the interference removal results of ANC. This is true as long as the low-rank signal assumption holds.

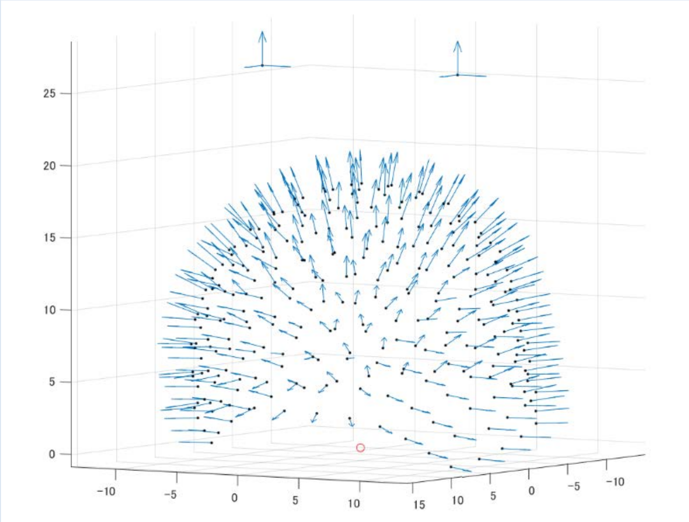

Fig.1: 256-channel CTF sensor array with six imaginary reference sensors. The arrows indicate the sensor orientations.

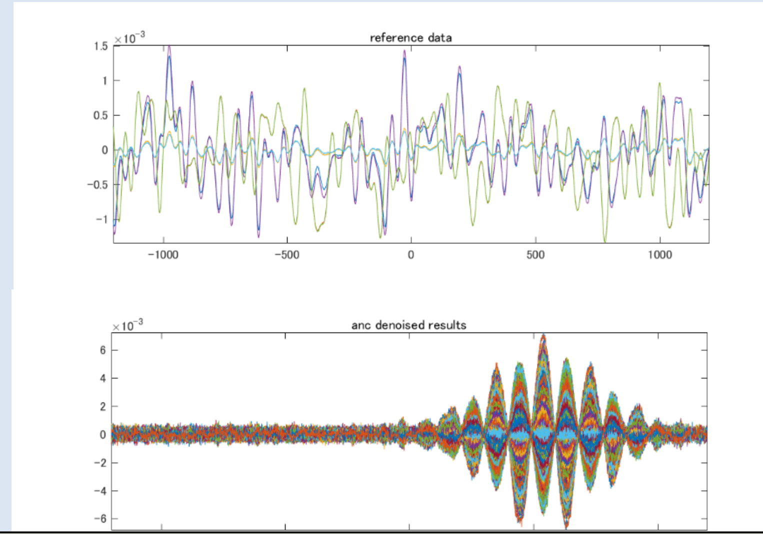

Fig.3: Reference-sensor time courses and results of ANC interference removal.

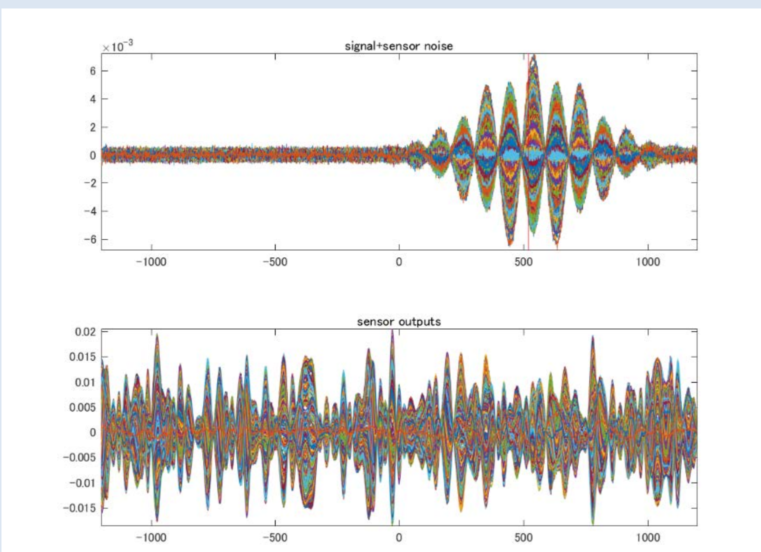

Fig.2: Signal time courses and interference overlapped sensor time courses.

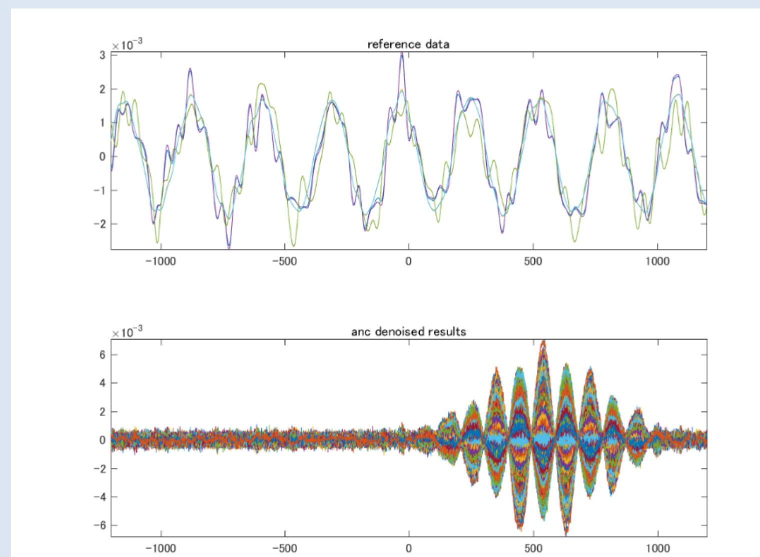

Fig.4: Reference-sensor time courses and the results of ANC interference removal when low frequency disturbance was added to the reference sensor data.

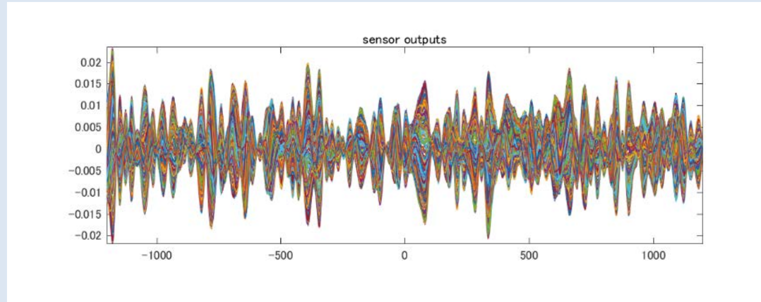

Fig.5: Interference overlapped sensor time courses. The interference was generated using six independent random activities.

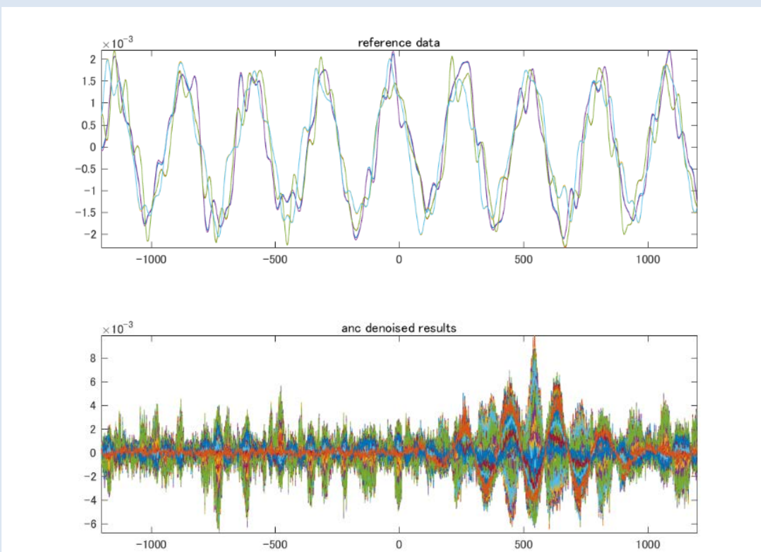

Fig.6: Reference-sensor time courses and the results of ANC interference removal when low frequency disturbance was added to the reference sensor data. The interference was generated using six independent random activities (Fig. 5).

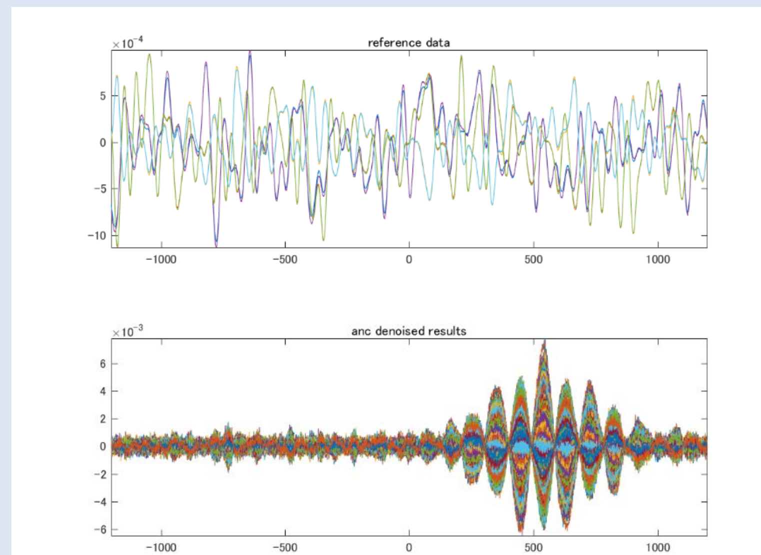

Fig.7: Reference-sensor time courses and the results of ANC interference removal when low frequency disturbance was not added to the reference sensor data. The interference was generated using six independent random activities (Fig. 5).