Neuromagnetic Source Reconstruction and Inverse Modeling

Part I: Introduction to adaptive spatial filter techniques

Kensuke Sekihara

Tokyo Metropolitan Institute of Technology
This talk:

• formulates the neuromagnetic source reconstruction problem using spatial filters.

• introduces non-adaptive and adaptive spatial filter techniques.

• focuses on the adaptive spatial filter technique (adaptive beamformer).
Magnetoencephalography (Neuromagnetic measurements)

• can provide a high temporal resolution.

• cannot provide (adequate) information on the source spatial configuration.

Efficient numerical algorithms for estimating source configuration are needed to be developed.

(Source localization problems)
Source localization problem

- Dipole modeling approach
- Image reconstruction approach
  - Tomographic reconstruction
  - Spatial filter
Tomographic reconstruction

• Assume pixel grids in the region of interest.

• Assume a source at each grid.

• Estimate the moment of each source by least-squares fitting to the measured data.
Spatial filter technique

• Form spatial filter weight $w(r)$ that focuses the sensitivity of the sensor array at a small area at $r$.

• Scan this focused area over the region of interest to obtain source reconstruction.
Right posterior tibial nerve stimulation measured by a 37-channel sensor array

Hashimoto et al., NeuroReport 2001
Right median nerve stimulation
measured by a 160-channel whole-head sensor array

Hashimoto et al., J. Clinical Neurophysiology submitted for publication
Definitions

- data vector: \( \mathbf{b}(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_M(t) \end{bmatrix} \)

- data covariance matrix: \( \mathbf{R} = \langle \mathbf{b}(t)\mathbf{b}^\top(t) \rangle \)

- source magnitude: \( \mathbf{s}(r,t) \)

- source orientation: \( \mathbf{\eta}(r,t) = [\eta_x(r,t), \eta_y(r,t), \eta_z(r,t)]^\top \)
Lead field vector for the source orientation $\eta(r)$

$$L(r) = \begin{bmatrix}
    l_1^x(r) & l_1^y(r) & l_1^z(r) \\
    l_2^x(r) & l_2^y(r) & l_2^z(r) \\
    \vdots & \vdots & \vdots \\
    l_M^x(r) & l_M^y(r) & l_M^z(r)
\end{bmatrix}, \quad l(r) = L(r) \begin{bmatrix}
    \eta_x(r) \\
    \eta_y(r) \\
    \eta_z(r)
\end{bmatrix}$$

$$|s_x| = |s_y| = |s_z| = 1$$
Basic relationship

\[ b_j(t) = \int l_j(r)s(r,t)dr \]

or

\[ b(t) = \int L(r)s(r,t)dr \]

Problem of source localization:

Estimate \( s(r,t) \) from the measurement \( b(t) \)
Spatial filter

\[ \hat{s}(r, t) = w^T(r)b(t) = [w_1(r), \ldots, w_M(r)] \begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix} = \sum_{m=1}^{M} w_m(r)b_m(t) \]

\( \uparrow \) \quad \text{estimate of} \quad \hat{s}(r, t) \quad \uparrow \quad \text{weight vector}
How to evaluate an appropriateness of the weight?

\[ b = \int L(r) s(r) dr \]
\[ \hat{s}(r) = w^T(r) b \]
\[ \rightarrow \hat{s}(r) = \int w^T(r) L(r') s(r') dr' \]
\[ \mathbb{R}(r, r') \]

Resolution kernel

\[ \hat{s}(r) = \int \mathbb{R}(r, r') s(r') dr' \]

(neglecting the explicit time notation)
Non-adaptive weight

$w(r)$ is data independent

Adaptive weight

$w(r)$ is data dependent
Data-independent (non-adaptive) weight

minimum-norm estimate (Hamalainen and Ilmoniemi)

The weight $w(r)$ is obtained by

$$
\min \int [ \Re (r, r') - \delta (r - r')]^2 dr'
\Downarrow

w^T (r) = L^T (r) G^{-1}, \text{ where } G_{i,j} = \int I_i (r) I_j^T (r) dr
\quad \text{Gram matrix}
$$

Inverse solution: $\hat{s}(r) = L^T (r) G^{-1} b$

This is erroneous
Gram matrix $G$ is usually calculated by introducing pixel grid $r_j$

$$b = \int L(r)s(r)dr = \sum_{j=1}^{N} L(r_j)s(r_j)$$

$$= \begin{bmatrix} L(r_1), \cdots, L(r_N) \end{bmatrix} \begin{bmatrix} s(r_1) \\ \vdots \\ s(r_N) \end{bmatrix} = L_N s_N$$

Therefore $G = L_N L_N^T$ and

$$w^T(r) = L^T(r)(L_N L_N^T)^{-1}$$
Resolution kernel for non-adaptive (minimum-norm) method
One example

• Auditory-evoked field were measured using 148-channel whole-head sensor array (Magnes 2500).

**Stimulus:** 1-kHz pure tone applied to subject’s left ear
The number of pixels: 12940 points
The condition number of $G$: $\sim 10^9$
Property of the gram matrix $G$

$$G_{i,j} = \int l_i(r) l_j(r) dr$$

Biomagnetic instruments

Overlaps of sensor lead fields is large

$G$ is poorly conditioned

$G \approx$ unit matrix

X-ray computed tomography
$\mathbf{G}$ is poorly conditioned

- Apply regularization when calculating $\mathbf{G}^{-1}$

\[
\begin{align*}
\hat{s} = \mathbf{H} \mathbf{L}^T \left( \mathbf{L}_N \mathbf{H} \mathbf{L}_N^T + \gamma \mathbf{I} \right)^{-1} \mathbf{b} \\
\end{align*}
\]

- Baysian-type approach

\[
\min_{\hat{s}} F : F = \| \mathbf{b} - \mathbf{L} \hat{s} \|^2 + \| \mathbf{H} \hat{s} \|^2 
\]

- Do not use $\mathbf{G}$

- Adaptive beamforming technique

\[
\begin{align*}
\end{align*}
\]
Adaptive spatial filter

Minimum-variance beamformer

\[ \hat{s}(r,t) = w^T(r)b(t) = [w_1(r), \ldots, w_M(r)] \begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix} = \sum_{m=1}^{M} w_m(r)b_m(t) \]

weight vector

\[
\begin{align*}
\min_{w} w^T R w & \text{ subject to } w^T l(r) = 1 \quad \Rightarrow \quad w^T(r) = \frac{\bar{l}^T(r)R^{-1}l(r)}{\bar{l}^T(r)R^{-1}l(r)}
\end{align*}
\]

\[ \langle \hat{s}(r,t)^2 \rangle = \frac{1}{\bar{l}^T(r)R^{-1}l(r)} \]
Assumption that source activities are uncorrelated

With constraint: \( w^T (r_p) l(r_p) = 1 \)

\[
\begin{align*}
w^T (r_p) R w(r_p) &= \langle s(r_p, t)^2 \rangle + \sum_{q \neq p} \langle s(r_q, t)^2 \rangle \| w^T (r_p) l(r_q) \|_2 \\
\uparrow &\\
\langle s(r_p, t) s(r_q, t) \rangle &= 0 \text{ when } p \neq q
\end{align*}
\]

\[
\min_w [w^T (r_p) R w(r_p)] \Rightarrow w^T (r_p) l(r_q) = 0, \ q \neq p
\]

Therefore, this minimization gives the weight satisfying

\[
\begin{align*}w^T (r_p) l(r_q) &= 1 \text{ for } p = q \\
&= 0 \text{ for } p \neq q
\end{align*}
\]
Spatial filter technique

• Form spatial filter weight $w(r)$ that focuses the sensitivity of the sensor array at a small area at $r$.  

Focused region
Adaptive beamformer sensitivity pattern: plot of $w(r_0)l(r)$

The density of the colors is proportional to $w(r_0)l(r)$.

The weight sets null-sensitivity at regions where sources exist.
Low-rank signal assumption

Consider a easiest case where we know locations and orientations of all $Q$ sources

weight $\mathbf{w}(r_1)$ (containing $M$ unknowns) can be obtained by solving a set of $Q$ linear equations:

\[
\mathbf{w}^T (r_1) \mathbf{l}(r_1) = \mathbf{w}_1(r_1) \mathbf{l}_1(r_1) + \ldots + \mathbf{w}_M(r_1) \mathbf{l}_M(r_1) = 1
\]

\[
\mathbf{w}^T (r_1) \mathbf{l}(r_2) = \mathbf{w}_1(r_1) \mathbf{l}_1(r_2) + \ldots + \mathbf{w}_M(r_1) \mathbf{l}_M(r_2) = 0
\]

\[
\vdots
\]

\[
\mathbf{w}^T (r_1) \mathbf{l}(r_Q) = \mathbf{w}_1(r_1) \mathbf{l}_1(r_Q) + \ldots + \mathbf{w}_M(r_1) \mathbf{l}_M(r_Q) = 0
\]

when $Q > M$, there is no solution for $\mathbf{w}^T (r_1)$
Low-rank signal

Number of sensors $M > \text{Number of sources } Q$

$$R = U \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_M \end{bmatrix} U^T = U \begin{bmatrix} \Lambda_s & 0 \\ 0 & \Lambda_N \end{bmatrix} U^T$$

$$U = [e_1, \ldots, e_Q \mid e_{Q+1}, \ldots, e_M] = [E_S \mid E_N]$$

$$\Gamma_s^{-1} = E_S \Lambda_s^{-1} E_S^T \text{ and } \Gamma_N^{-1} = E_N \Lambda_N^{-1} E_N^T \Rightarrow R^{-1} = \Gamma_s^{-1} + \Gamma_N^{-1}$$
Orthogonality principle

\[ E_N^T l(r_q) = \Gamma_N^{-1} l(r_q) = 0 \] at any source location \( r_q \)

Minimum-variance spatial filter output:

\[
\langle \hat{S}(r)^2 \rangle = \frac{1}{l^T (r) R^{-1} l(r)} = \frac{1}{l^T (r) \Gamma_S^{-1} l(r) + l^T (r) \Gamma_N^{-1} l(r)}
\]
148-channel sensor array
Minimum-norm reconstruction

Minimum-variance spatial filter reconstruction

MUSIC reconstruction
Adaptive-beamformer techniques were originally developed in the fields of array signal processing, including radar, sonar, and seismic exploration.

Two major problems arise when applying minimum-variance beamformer to MEG source localization.

(1) Vector source detection.

(2) Output SNR degradation.
Vector source detection

The neuromagnetic sources are three dimensional vectors.

⇒

The minimum-variance beamformer formulation should be extended to incorporate the vector nature of sources.

Two-types of extensions has been proposed: scalar and vector formulations.
Scalar MV beamformer formulation

\[ w^\top (r, \eta) = \frac{l^\top (r, \eta) R^{-1}}{l^\top (r, \eta) R^{-1} l (r, \eta)} = \frac{\eta^\top L^\top (r) R^{-1}}{\eta^\top L^\top (r) R^{-1} L (r) \eta} \]

The weight depends not only on \( r \) but also on \( \eta \).

Vector MV beamformer formulation

\[ [w_x (r), w_y (r), w_z (r)]^\top = [L^\top (r) R^{-1} L (r)]^{-1} L^\top (r) R^{-1} \]

The three weight vectors detect \( x, y, \) and \( z \) components. Calculation of the weight does not require \( \eta \).
Scalar MV beamformer formulation

\[
\begin{align*}
\mathbf{w}^\top (r, \eta) &= \frac{\mathbf{l}^\top (r, \eta) \mathbf{R}^{-1} \mathbf{L}^\top (r) \mathbf{R}^{-1} \mathbf{L}(r) \mathbf{\eta}}{\mathbf{l}^\top (r, \eta) \mathbf{R}^{-1} \mathbf{l}(r, \eta) \mathbf{L}(r) \mathbf{\eta}} = \\
&= \frac{\eta^\top \mathbf{L}^\top (r) \mathbf{R}^{-1} \mathbf{L}(r) \mathbf{\eta}}{\eta^\top \mathbf{L}^\top (r) \mathbf{R}^{-1} \mathbf{L}(r) \mathbf{\eta}}
\end{align*}
\]

The weight depends not only on \( r \) but also on \( \eta \).

Vector MV beamformer formulation

\[
[\mathbf{w}_x(r), \mathbf{w}_y(r), \mathbf{w}_z(r)]^\top = [\mathbf{L}^\top (r) \mathbf{R}^{-1} \mathbf{L}(r)]^{-1} \mathbf{L}^\top (r) \mathbf{R}^{-1}
\]

The three weight vectors detect \( x, y, \) and \( z \) components. Calculation of the weight does not require \( \eta \).
How to derive the optimum $\eta$ in scalar formulation?

\[ \max_{\eta} \langle \hat{S}(r,t)^2 \rangle = \max_{\eta} \frac{1}{\eta^T L^T (r) R^{-1} L(r) \eta} = \left[ \min_{\eta} (\eta^T L^T (r) R^{-1} L(r) \eta) \right]^{-1} \]

Eigendecomposition: $[L^T (r) R^{-1} L(r)]_{j=1}^3 = \sum_{j=1}^3 \gamma_j v_j v_j^T$, $(\gamma_1 > \gamma_2 > \gamma_3)$

\[ \min_{\eta} (\eta^T L^T (r) R^{-1} L(r) \eta) = \gamma_3 \]

minimum eigenvalue

\[ \max_{\eta} \langle \hat{S}(r,t)^2 \rangle = \frac{1}{\gamma_3} = S_{opt} \]
Vector beamformer formulation

Each weight vector, \( w_x(r), w_y(r), \) or \( w_z(r) \) is obtained by using the following multiple constraints.

\[
\begin{align*}
\min w_x^T R w_x \quad & \text{subject to } w_x^T l(r, e_x) = 1, \ w_x^T l(r, e_y) = 0, \ w_x^T l(r, e_z) = 0 \\
\min w_y^T R w_y \quad & \text{subject to } w_y^T l(r, e_x) = 0, \ w_y^T l(r, e_y) = 1, \ w_y^T l(r, e_z) = 0 \\
\min w_z^T R w_z \quad & \text{subject to } w_z^T l(r, e_x) = 0, \ w_z^T l(r, e_y) = 0, \ w_z^T l(r, e_z) = 1
\end{align*}
\]

\( e_x, e_y, e_z \) : unit vectors in the \( x, y, z \) directions.

\[
\downarrow
\]

\[
[w_x(r), w_y(r), w_z(r)]^T = [L^T(r)R^{-1}L(r)]^{-1}L^T(r)R^{-1}
\]

(The weight is not equal to the scalar weight with \( \eta = e_x, e_y, \) or \( e_z \)).

van Veen et al., 1996   Spencer et al., 1992
In vector formulation, $\eta$ that gives the maximum power output can be obtained using

$$\max_{\eta} \left\langle \hat{s}(r, t)^2 \right\rangle = \max_{\eta} \left\| \begin{bmatrix} \hat{s}_x(r) \\ \hat{s}_y(r) \\ \hat{s}_z(r) \end{bmatrix} \right\|^2 = \max_{\eta} \eta^T \begin{bmatrix} L^T(r)R^{-1}L(r) \end{bmatrix}^{-1} \eta$$

\[\downarrow\]

$$\max_{\eta} \left\langle \hat{s}(r, t)^2 \right\rangle = 1 / \gamma_3 = S_{opt}$$

Either types of formulations attain $S_{opt}$ when the beamformer pointing direction is optimized.

\[\downarrow\]

Two types of formulations are mathematically equivalent.
The image shows a 148-channel sensor array with a reconstructed region highlighted. The sensor array is labeled with points for the 1st, 2nd, and 3rd channels. Two graphs are presented: one for a scalar formulation and another for a vector formulation. The scalar formulation graph shows a contour plot with labels for y and z coordinates, while the vector formulation graph also includes these coordinates. The scalar formulation graph appears to show a distribution of data points or fields in the reconstruction region, whereas the vector formulation graph likely represents vector quantities in the same region.
Minimum-variance beamformer is very sensitive to errors in forward modeling or errors in sample covariance matrix. Therefore, because such errors are almost inevitable in neuromagnetic measurements, minimum-variance beamformer generally provides noisy spatio-temporal reconstruction results.

Introducing eigenspace projection.
Add very small amount of noise to obtain the simulated MEG recordings
Spatio-temporal reconstruction
Recall some definitions:

\[
R = U \begin{bmatrix}
\lambda_1 & 0 & \cdots & 0 \\
0 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & \lambda_M
\end{bmatrix} U^T = U \begin{bmatrix}
\Lambda_S & 0 \\
0 & \Lambda_N
\end{bmatrix} U^T, \quad \text{and} \quad U = [e_1, \ldots, e_p | e_{p+1}, \ldots, e_M]
\]

Also, \( \Gamma_S^{-1} = E_S \Lambda_S^{-1} E_S^T \), \( \Gamma_N^{-1} = E_N \Lambda_N^{-1} E_N^T \)

Output SNR \( \infty \) \[
\frac{[l^T(r) \Gamma_S^{-1} l(r)]^2}{[l^T(r) \Gamma_S^{-2} l(r)] + \epsilon^T \Gamma_N^{-2} \epsilon^*}
\]

overall error in estimating \( l(r) \)

Even when \( \epsilon \) is small, \( \epsilon^T \Gamma_N^{-2} \epsilon \) may not be small,
because \( \epsilon^T \Gamma_N^{-2} \epsilon \approx \| \epsilon \|^2 / \lambda_{p+j}^2 \leftarrow \text{noise level eigenvalue} \)
Eigenspace projection

The error term $\varepsilon^T \Gamma_N^{-2} \varepsilon$ arises from the noise subspace component of $w(r)$.

Extension to eigenspace projection beamformer

$$\overline{w}_\mu = E_S E_S^T w_\mu,$$  where $\mu = x, y$ or $z$

Output SNR $\propto \frac{[I^T(r) \Gamma_S^{-1} I(r)]^2}{[I^T(r) \Gamma_S^{-2} I(r) + \varepsilon^T \Gamma_N^{-2} \varepsilon]}$  (non-eigenspace projected)

$\Downarrow$

Output SNR $\propto \frac{[I^T(r) \Gamma_S^{-1} I(r)]^2}{[I^T(r) \Gamma_S^{-2} I(r) ]}$  (eigenspace projected)
Spatio-temporal reconstruction with eigen-space projection
Application to 37-channel auditory-somatosensory recording
eigenspace-projection results
Application to 37-channel auditory-somatosensory recording
Non-eigenspace projected results
Application to 37-channel auditory-somatosensory recording
eigenspace-projection results
Summary

• This talk reviews the application of adaptive beamformer to reconstruction of brain activities, with some reference to the non-adaptive methods.

• Two types of extensions, scalar and vector extensions to incorporate the vector nature of the sources, are shown to be mathematically equivalent.

• Eigenspace projection is shown to overcome the SNR degradation problem.

• The implicit assumptions for the adaptive beamformer are that sources are uncorrelated and that the signal is low rank. The cases where these assumptions are invalidated will be discussed in Part II.
Summary

• This talk reviews the application of adaptive beamformer to reconstruction of brain activities, with some reference to the non-adaptive methods.

• Two types of extensions, scalar and vector extensions to incorporate the vector nature of the sources, are shown to be mathematically equivalent.

• Eigenspace projection is shown to overcome the SNR degradation problem.

• The implicit assumptions for the adaptive beamformer are that sources are uncorrelated and that the signal is low rank. The cases where these assumptions are invalidated will be discussed in Part II.
Summary

• This talk reviews the application of adaptive beamformer to reconstruction of brain activities, with some reference to the non-adaptive methods.

• Two types of extensions, scalar and vector extensions to incorporate the vector nature of the sources, are shown to be mathematically equivalent.

• Eigenspace projection is shown to overcome the SNR degradation problem.

• The implicit assumptions for the adaptive beamformer are that sources are uncorrelated and that the signal is low rank. The cases where these assumptions are invalidated will be discussed in Part II.
Summary

• This talk reviews the application of adaptive beamformer to reconstruction of brain activities, with some reference to the non-adaptive methods.

• Two types of extensions, scalar and vector extensions to incorporate the vector nature of the sources, are shown to be mathematically equivalent.

• Eigenspace projection is shown to overcome the SNR degradation problem.

• The implicit assumptions for the adaptive beamformer are that sources are uncorrelated and that the signal is low rank. The cases where these assumptions are invalidated will be discussed in Part II.
Collaborators

University of California, San Francisco
Biomagnetic Imaging Laboratory
  Dr. Srikantan S. Nagarajan

University of Maryland
Linguistics and Cognitive Neuroscience Laboratory
  Dr. David Poeppel

Massachusetts Institute of Technology,
Department of Linguistics and Philosophy
  Dr. Alec Marantz

Kanazawa Institute of Technology
Human Science Laboratory
  Dr. Isao Hashimoto
Visit

http://www.tmit.ac.jp/~sekihara/

The PDF version of this power-point presentation as well as PDFs of the recent publications are available.