Prewhitening Signal Covariance Estimation and Prewhitening Beamforming

Kensuke Sekihara\textsuperscript{1}, Kenneth E. Hild II\textsuperscript{2}, Srikantan S. Nagarajan\textsuperscript{2}

\textsuperscript{1}Department of Systems Design & Engineering, Tokyo Metropolitan University
\textsuperscript{2}Biomagnetic Imaging Laboratory, University of California, San Francisco
Data model

\[ \mathbf{b}(t) = \mathbf{b}_S(t) + \mathbf{b}_I(t) + \mathbf{n}(t) \]

- Measured magnetic field
- Background spontaneous activity
- Sensor noise
- Signal magnetic field

Definitions

Data covariance: \( \mathbf{R} = \langle \mathbf{b}(t)\mathbf{b}^T(t) \rangle \)

Signal covariance: \( \mathbf{R}_S = \langle \mathbf{b}_S(t)\mathbf{b}_S^T(t) \rangle \)

Interference plus noise covariance: \( \mathbf{R}_{i+n} = \langle (\mathbf{b}_I(t) + \mathbf{n}(t))(\mathbf{b}_I(t) + \mathbf{n}(t))^T \rangle \)
Two-condition experiments (ideal scenario)

Task: \( b(t) = b_s(t) + b_I(t) + n(t) \)

Control: \( b_C(t) = b_I(t) + n(t) \)

Covariance matrix relations

Task: \( R = R_s + R_{i+n} \)

Control: \( R_C = R_{i+n} \)

Problem:

How to obtain interference-free source reconstruction

How to estimate \( R_s \) using \( R \) and \( R_C \)
Existing approach: image-based subtraction (with $t$ test)

Define $s(r)$: source reconstruction from $b(t)$
$s_c(r)$: source reconstruction from $b_c(t)$

Calculate $\Delta s(r) = s(r) - s_c(r)$

This approach works well when SIR is high but becomes less effective when large interference exists.

Naive approach: covariance-based subtraction

Estimate $R_S$ using $\hat{R}_S = R - R_C$

This approach has many problems, and generally does not work well
Prewhitening estimation of signal covariance

Calculate $\tilde{R} = R_{C}^{-1/2} R_{R} R_{C}^{-1/2}$
(Tilde is used to indicate the prewhitened version of a matrix)

$$\tilde{R}_{S} = \sum_{j=1}^{Q} \gamma_{j} u_{j} u_{j}^{T} \implies \tilde{R} = \sum_{j=1}^{Q} (\gamma_{j} + 1) u_{j} u_{j}^{T} + \sum_{j=Q+1}^{M} u_{j} u_{j}^{T}.\uparrow$$

$\tilde{R}$ has signal-level eigenvalues greater than 1, and their eigenvectors are equal to those of $\tilde{R}_{S}$

Signal covariance estimation

$$\hat{R}_{S} = R_{C}^{1/2} \left[ U_{S} U_{S}^{T} (\tilde{R} - I) \right] R_{C}^{1/2} = R_{C}^{1/2} \left[ \sum_{j=1}^{Q} (\gamma_{j} - 1) u_{j} \right] R_{C}^{1/2} \uparrow$$

$U_{S} = [u_{1}, \ldots, u_{Q}]$
A total 96 of these paired data sets were generated, and used for calculating $R$ and $R_c$. 

Computer simulation on two-condition multi-epoch measurements
Two-condition measurements-Computer Simulation

Signal-to-Interference Ratio: 0.5

Relative source intensity

<table>
<thead>
<tr>
<th>Source</th>
<th>Control</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>First source</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Second source</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Third source</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Minimum-variance source reconstruction

\[
\hat{s}(r, t) = \frac{1}{[l^T(r)R^{-1}l(r)]}
\]

\[
\hat{s}_{PW}(r, t) = \frac{1}{[l^T(r)(\hat{R}_S + \mu I)^{-1}l(r)]}
\]

Source location
Two-condition experiments: more realistic scenario
- Scenario I

Some sources exist only in the control condition

Task: \[ b(t) = b_s(t) + b_I(t) + n(t) \]

Control: \[ b_C(t) = b_I(t) + n(t) + b_\Delta(t) \]

Signal from control-only sources

\[ \uparrow \]

Task: \[ R = R_S + R_{i+n} \]

Control: \[ R_C = R_{i+n} + R_\Delta \]

covariance from control-only source: \[ R_\Delta = \left\langle b_\Delta(t) b_\Delta^T(t) \right\rangle \]
Scenario I-analysis

Define
\[ \tilde{R}_S = \sum_{j=1}^{Q} \gamma_j u_j u_j^T, \quad \text{and} \quad \tilde{R}_\Delta = \sum_{j=1}^{P} \beta_j v_j v_j^T \]

Then
\[ \tilde{R} = \tilde{R}_S + I - \tilde{R}_\Delta = \sum_{j=1}^{Q} \gamma_j u_j u_j^T + \sum_{j=P+1}^{M} \beta_j v_j v_j^T + \sum_{j=1}^{P} (1 - \beta_j) v_j v_j^T \]

Thus
\[ \tilde{R} u_j = (\sum_{j=1}^{Q} \gamma_j u_j u_j^T) u_j + (\sum_{j=P+1}^{M} \beta_j v_j v_j^T) u_j + (\sum_{j=1}^{P} (1 - \beta_j) v_j v_j^T) u_j \]
\[ = \gamma_j u_j + \sum_{j=1}^{P} (1 - \beta_j) (v_j^T u_j) v_j \]

Prewhitening estimation \( \hat{R}_S = R_C^{1/2} \left[ U_S U_S^T (\tilde{R} - I) \right] R_C^{1/2} \) is still effective if \( \text{span}(u_1, u_2, \ldots, u_Q) \perp \text{span}(v_1, v_2, \ldots, v_P) \) approximately holds.
Two-condition experiments: more realistic scenario  
- Scenario II

Target signal sources are active also in the control condition  
(Their intensities change between the two conditions)

\[
\text{Task: } \quad b(t) = b_s(t) + b_i(t) + n(t)
\]

\[
\text{Control: } \quad b_c(t) = b_s(t)' + b_i(t) + n(t)
\]

\[
\text{Task: } \quad R = R_s + R_{i+n}
\]

\[
\text{Control: } \quad R_c = R_s' + R_{i+n}
\]

\[
R_s' = \langle b_s(t)' b_s^T(t)' \rangle
\]
Scenario II-analysis

We finally have

\[
\tilde{R} = \tilde{D}_P + I - \tilde{D}_N
\]

where

\[D_P : \text{covariance matrix from signal sources stronger in the task than in the control state.}\]

\[D_N : \text{covariance matrix from signal sources weaker in the task than in the control state.}\]

The above covariance relationship is the same as that for Scenario I

We can still use \[
\hat{D}_P = R_{C}^{1/2} \left[ U_S U_S^T (\tilde{R} - I) \right] R_{C}^{1/2}
\]

\[\hat{D}_N \] is obtained from flipped prewhitening where \[
\tilde{R}_C = R^{-1/2} R_C R^{-1/2}
\]
is used.
Scenario I and II--Numerical Experiments

Signal-to-Interference Ratio: 0.5

Relative source intensity

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<td>1</td>
</tr>
<tr>
<td>Third source</td>
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<td>1</td>
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Task

Control

Prewhitening

Flipped

Subtracted

First

Second

Third
Scenario I and II--Numerical Experiments

Signal-to-Interference Ratio: 0.25

Relative source intensity

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Task

Control

Subtracted

Prewhitening

Flipped
Robustness to overestimation of signal subspace dimension

Signal-to-Interference Ratio: 0.5

Eigenvalue spectrum of $\tilde{R}$

Prewhitening

Flipped

$Q = 10$

$Q = 20$

$Q = 50$
Overestimation of signal-subspace dimension

\[
\hat{R}_S = R_C^{1/2} \left[ (U_S U_S^T + U_\varepsilon U_\varepsilon^T) (\tilde{R} - I) \right] R_C^{1/2} = R_S + \Delta R_S
\]

Overestimated term

Error

\[
\Delta R_S = \sum_{j=1}^{Q} \Delta \lambda_j e_j e_j^T + \sum_{j=Q+1}^{M} \Delta \mu_j e_j e_j^T
\]

signal subspace component

noise subspace component

\[
\hat{R}_{S+n} = R_S + \Delta R_S + \mu I = \sum_{j=1}^{Q} (\lambda_j + \Delta \lambda_j) e_j e_j^T + \sum_{j=Q+1}^{M} (\mu + \Delta \mu_j) e_j e_j^T
\]

These terms modify source intensity

These terms modify regularization constant
Hand-motor measurement

Voluntary right finger movement (every 5 sec)

Calculate $R_c$

-0.3

0

1.0

1.3 sec

Calculate $R$

Typical raw epoch

Power spectrum

MRI overlay

Prewhitening results

Subtraction results
Hand-motor measurement

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<th>Calculate $R$</th>
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</thead>
<tbody>
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<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.3 sec</td>
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Voluntary right finger movement (every 5 sec)

Typical raw epoch

Power spectrum

MRI overlay

Prewhitening results

Subtraction results
Summary

• We propose a novel prewhitening signal covariance estimation method.

• The proposed prewhitening method is shown to be effective in more realistic (non-ideal) scenarios of two-condition experiments.

• Prewhitening method gives significantly better source reconstruction (less bias and higher spatial resolution) than the existing subtraction-based method.

This investigation is presented at poster P-163, (session P4-1) 8/22 3:00—5:00PM
Collaborators

Kenneth E. Hilde
Sarang S. Dilal
Johanna M. Zumer
Hagai Attias
Susanne Honma
Anne Findlay
Mary Mantle

Thank you for your attention

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