# Dual signal subspace projection (DSSP) algorithm

Kensuke Sekihara[1,2] and Srikantan S. Nagarajan[3]

[1]Tokyo Medical and Dental University

[2]Signal Analysis Inc.

[3]University of California, San Francisco

I am talking about the dual signal subspace projection (DSSP) algorithm. This algorithm can remove interference an order-of-magnitude greater than the signal without requiring separate noise measurements.

My talk:

- reviews the spatial-domain and  time-domain signal subspaces,
- explains the DSSP algorithm,
- presents results of applications on removal of VNS artifacts,
- compares the DSSP algorithm with the tSSS algorithm.

# Conventional (spatial-domain) signal subspace

Data model: $\boldsymbol{y}(t) = \boldsymbol{y}_S(t) + \boldsymbol{\varepsilon}$

data vector signal vector noise vector

**Key relationship:** $\boldsymbol{y}_S(t) = \sum_{q=1}^{Q} s_q(t)\boldsymbol{l}_q$

lead field vector at the $q$-th source location and orientation.

**Signal vector is expressed as the linear combination of these lead field vectors.**

$$\boldsymbol{y}_S(t) \in span\{\boldsymbol{l}_1, \ldots, \boldsymbol{l}_Q\} = E_S$$

Signal vector belongs to the span of these lead field vectors, which is defined as the signal subspace $E_S$ .

**We can define the signal subspace in the time-domain as well.**

# Time-domain signal subspace

We assume time-series measurement at time samples: $t_1, \ldots, t_K$

Time course of the *q*-th source: $s_q(t_1), \ldots, s_q(t_K)$

Source time-course (row) vector: $\boldsymbol{s}_q = [s_q(t_1), \ldots, s_q(t_K)]$

$span\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_Q\}$ can be defined as the time-domain signal subspace,

which is denoted as $K_S$ : $K_S = span\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_Q\}$ .

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Sekihara, K., & Nagarajan, S. S. (2017). Subspace-based interference removal methods for a multichannel biomagnetic sensor array. *Journal of Neural Engineering*, *14*(5), 051001.

## Let us introduce matrix-based representation.

Data are measured at $K$ time samples: $t_1, t_2, \ldots, t_K$

Data matrix: $\boldsymbol{B} = [\boldsymbol{y}(t_1), \ldots, \boldsymbol{y}(t_K)]$

Signal matrix: $\boldsymbol{B}_S = [\boldsymbol{y}_S(t_1), \ldots, \boldsymbol{y}_S(t_K)]$

**Data model**

vector representation

$$\boldsymbol{y}(t) = \boldsymbol{y}_S(t) + \boldsymbol{\varepsilon}$$

$\Rightarrow$

matrix representation

$$\boldsymbol{B} = \boldsymbol{B}_S + \boldsymbol{B}_{\boldsymbol{\varepsilon}}$$

**We have symmetric relationships between time- and spatial-domain signal subspaces.**

Each column of $\boldsymbol{B}_S \in E_S$

Each row of $\boldsymbol{B}_S \in K_S$

Column space of $\boldsymbol{B}_S = E_S$

Row space of $\boldsymbol{B}_S = K_S$

Signal space projection (SSP) is a well known algorithm for interference removal.

## There are two-types of SSP algorithms

Data model: $\boldsymbol{B} = \boldsymbol{B}_S + \boldsymbol{B}_I + \boldsymbol{B}_\varepsilon$

### Spatial-domain SSP

Compute a projector $\boldsymbol{P}_I$ from the column space of $\boldsymbol{B}_I$ .

Compute $(\boldsymbol{I} - \boldsymbol{P}_I)\boldsymbol{B}$ to remove $\boldsymbol{B}_I$ .

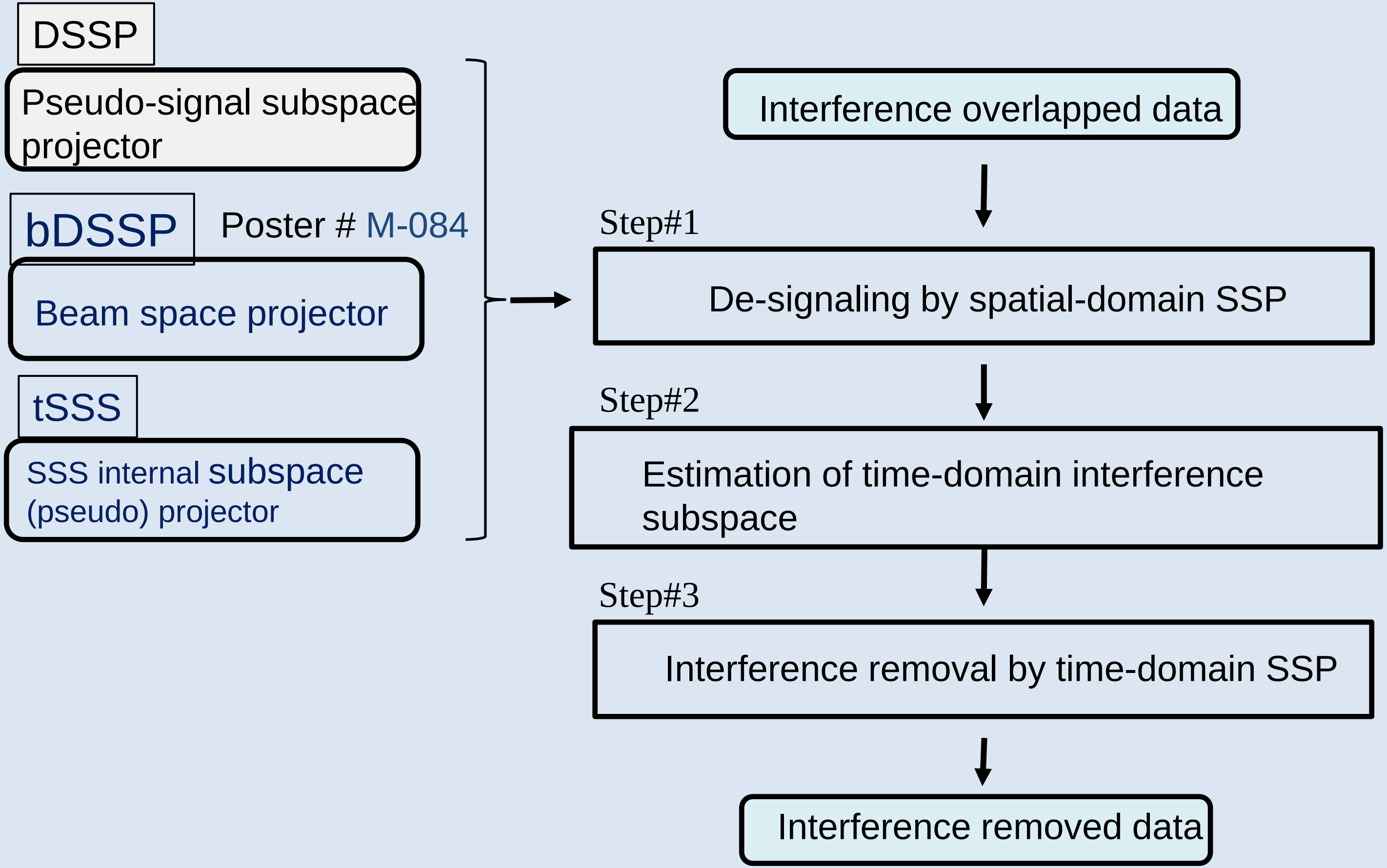
### Time-domain SSP

Compute a projector $\boldsymbol{\Pi}_I$ from the row space of $\boldsymbol{B}_I$ .

Compute $\boldsymbol{B}(\boldsymbol{I} - \boldsymbol{\Pi}_I)$ to remove $\boldsymbol{B}_I$ .

**DSSP makes use of these two types of SSPs**

# Structure of the DSSP algorithm



## Step#1: De-signaling (signal nulling)

Remove signal components from the data: $\boldsymbol{B} = \boldsymbol{B}_S + \boldsymbol{B}_I + \boldsymbol{B}_\varepsilon$

If the projector onto the signal subspace $\boldsymbol{P}_S$ is known, de-signaling is attained by spatial domain SSP, such that:

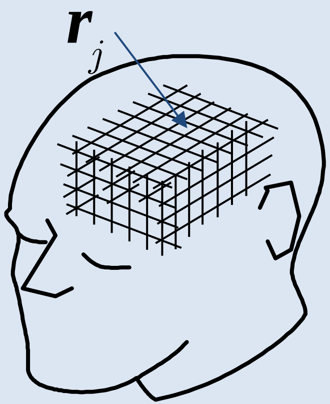
$$
\begin{aligned}
(\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B} &= (\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B}_S + (\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B}_I + (\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B}_\varepsilon \\
&= (\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B}_I + (\boldsymbol{I} - \boldsymbol{P}_S)\boldsymbol{B}_\varepsilon
\end{aligned}
$$

However, because $\boldsymbol{P}_S$ is unknown, we must use something that can substitute $\boldsymbol{P}_S$ .

⇩

DSSP uses the pseudo-signal subspace projector.

# Pseudo-signal subspace projector




## Augmented voxel lead field matrix over source space

$$\boldsymbol{L}_V = \left[ \boldsymbol{L}(\boldsymbol{r}_1), \ldots, \boldsymbol{L}(\boldsymbol{r}_N) \right]$$

**SVD of $\boldsymbol{L}_V$**

$$\boldsymbol{L}_V = \boldsymbol{U}\boldsymbol{\Gamma}\boldsymbol{V}^T = \left[ \boldsymbol{u}_1, \ldots, \boldsymbol{u}_M \right] \begin{bmatrix} \gamma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_M \end{bmatrix} \boldsymbol{V}^T$$

$span\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_\xi\}$ approximates the signal subspace, where $\gamma_1, \ldots, \gamma_\xi$ are leading $\xi$ singular values.

Defining the pseudo-signal subspace projector as $\tilde{\boldsymbol{P}} = \left[ \boldsymbol{u}_1, \ldots, \boldsymbol{u}_\xi \right] \left[ \boldsymbol{u}_1, \ldots, \boldsymbol{u}_\xi \right]^T$,

relationship: $\tilde{\boldsymbol{P}}\boldsymbol{B}_S = \boldsymbol{B}_S$ approximately holds, and de-signaling $(\boldsymbol{I} - \tilde{\boldsymbol{P}})\boldsymbol{B}_S = 0$ can be achieved.

# Step#2: Estimation of time-domain interference subspace

De-signaling step creates two kinds of these modified data matrices:

$$\tilde{\boldsymbol{P}}\boldsymbol{B} = \tilde{\boldsymbol{P}}\boldsymbol{B}_S + \tilde{\boldsymbol{P}}\boldsymbol{B}_I + \tilde{\boldsymbol{P}}\boldsymbol{B}_\varepsilon$$

$$(\boldsymbol{I} - \tilde{\boldsymbol{P}})\boldsymbol{B} = (\boldsymbol{I} - \tilde{\boldsymbol{P}})\boldsymbol{B}_I + (\boldsymbol{I} - \tilde{\boldsymbol{P}})\boldsymbol{B}_\varepsilon$$

We thus have these relationships:

Row space of $\tilde{\boldsymbol{P}}\boldsymbol{B} \subset K_S + K_I + K_\varepsilon$

Row space of $(\boldsymbol{I} - \tilde{\boldsymbol{P}})\boldsymbol{B} \subset K_I + K'_\varepsilon$

**Time-domain interference subspace is the common subspace.**

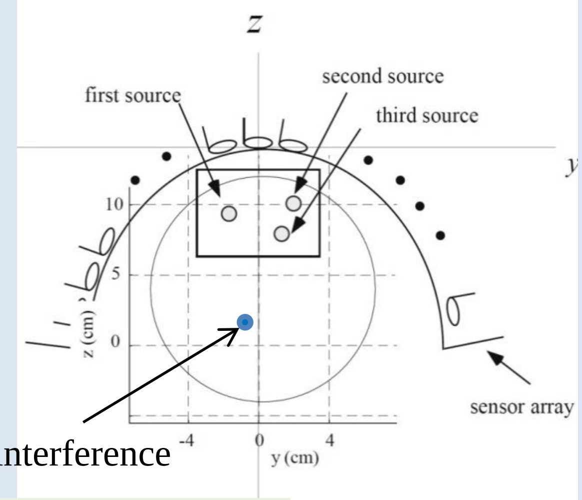
After some mathematical arguments, we can show:

$$K_I \approx (\text{row space of } \tilde{\boldsymbol{P}}\boldsymbol{B}) \cap (\text{row space of } (\boldsymbol{I} - \tilde{\boldsymbol{P}})\boldsymbol{B})$$

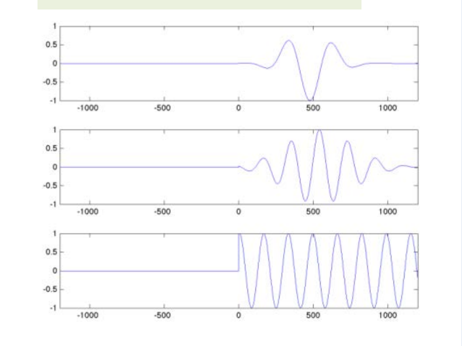
The basis vectors of $K_I$ are obtained as the basis vectors of this intersection.

**Time-domain SSP is then implemented for interference removal in Step#3.**

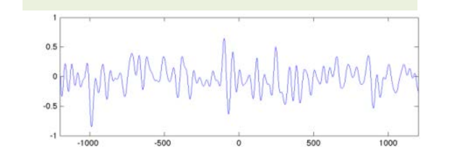
# Computer simulation




Source time courses



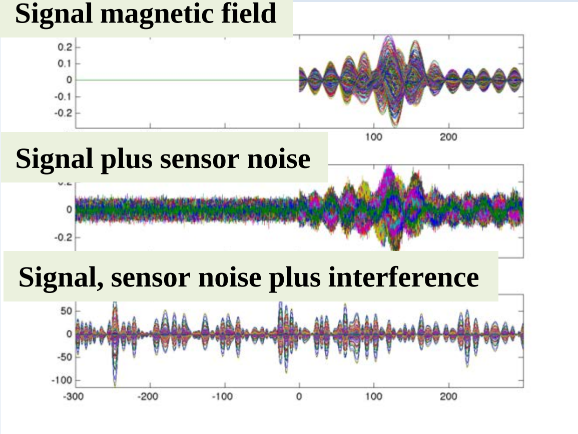

Interference time courses




**Signal magnetic field**

**Signal plus sensor noise**

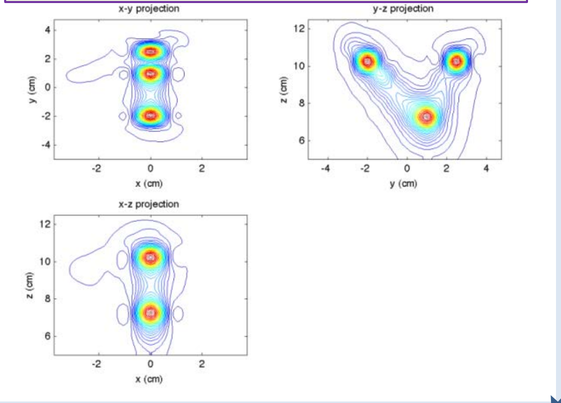
**Signal, sensor noise plus interference**




- 275 CTF sensor array
- signal to sensor noise ratio:4
- Interference to signal ratio: 100

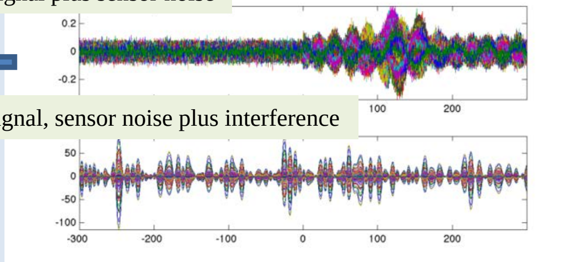
# Results of source localization

**Source image with no interference: Ground truth**

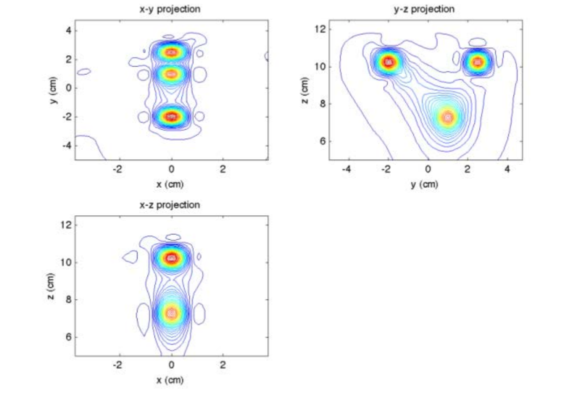



Signal plus sensor noise

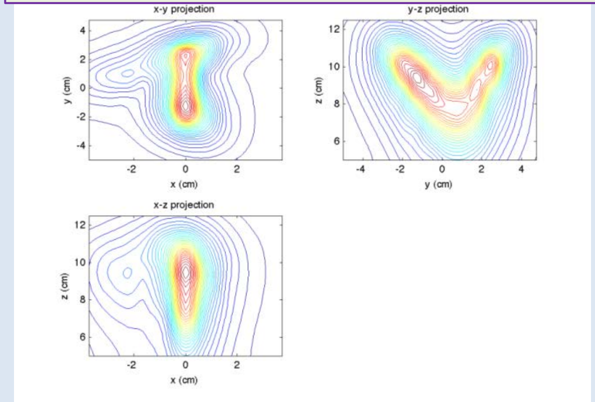
Signal, sensor noise plus interference



**Source image from DSSP results**




**Source image from interference-overlapped sensor data**



# Somatosensory data from a patient with a VNS device

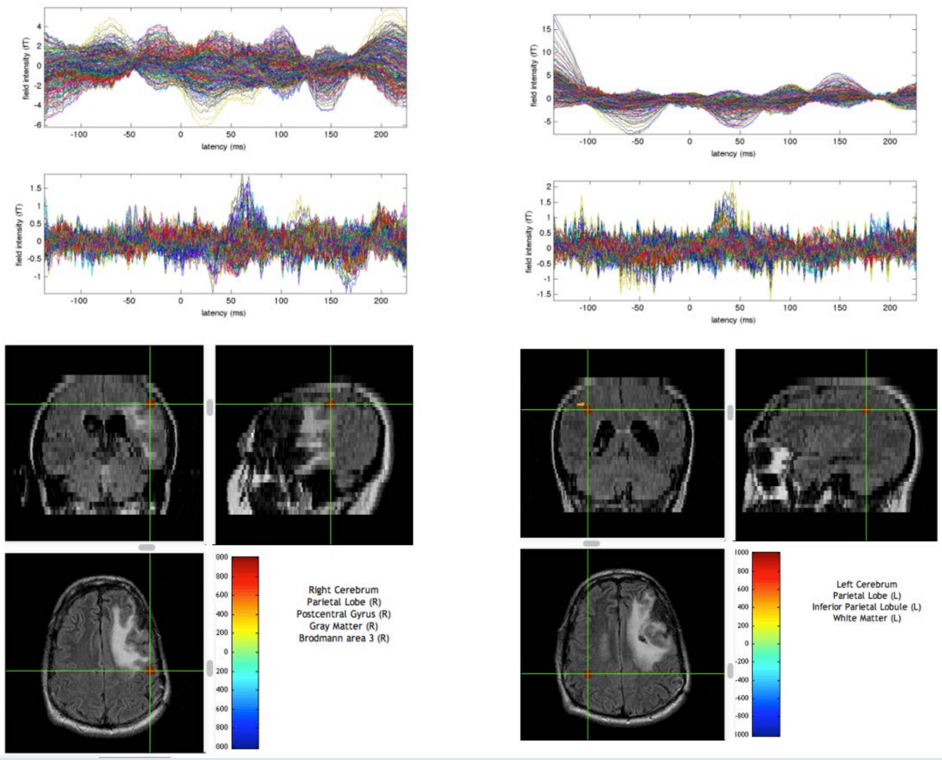
LD1 tactile stimulation

RD1 tactile stimulation

Original sensor data

DSSP results

Source image from DSSP results



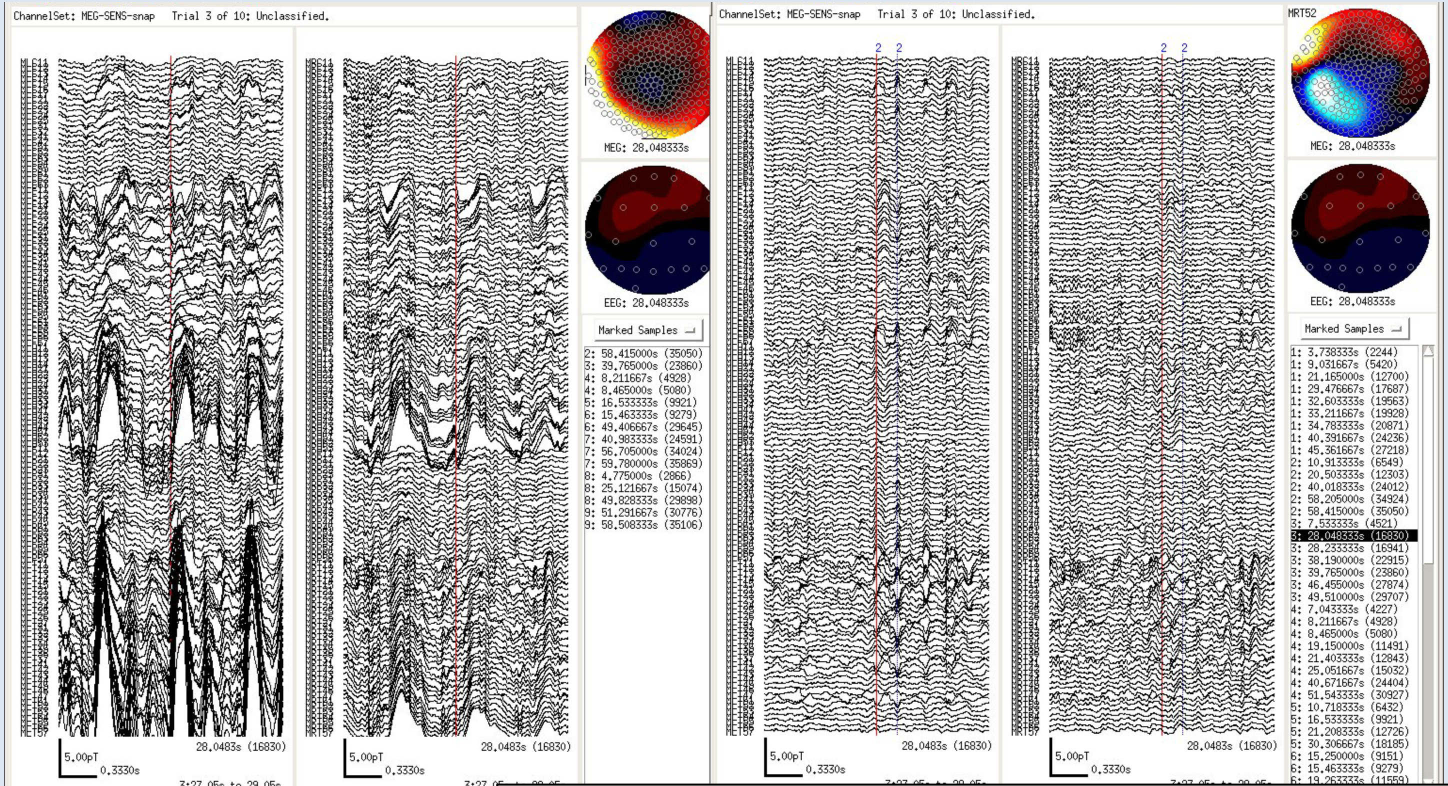

Contralateral SI cortices were reconstructed from DSSP results.

Without DSSP, a single strong fake source was reconstructed near the center of the head for the both cases.

# Removal of VNS artifacts from epilepsy data

**Original** | **Cleaned**

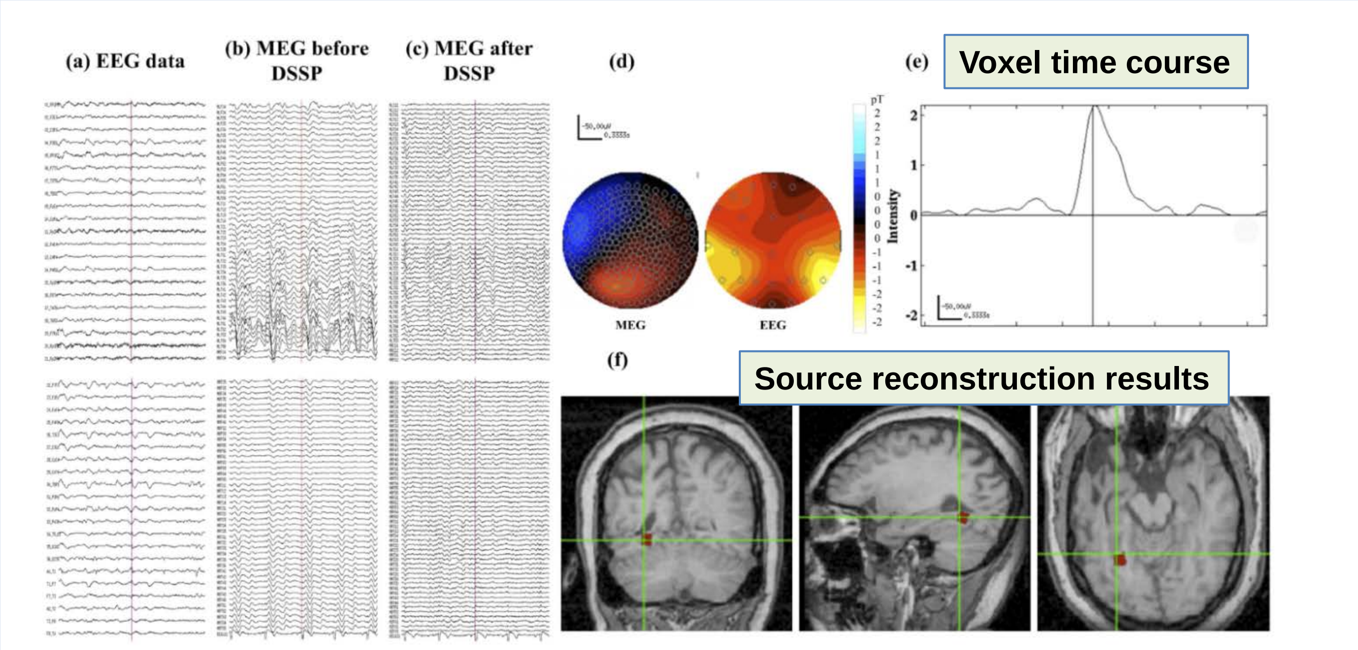
**MEG Left** **MEG Right** | **MEG Left** **MEG Right**




An epileptologist can identify several spikes in these cleaned data.

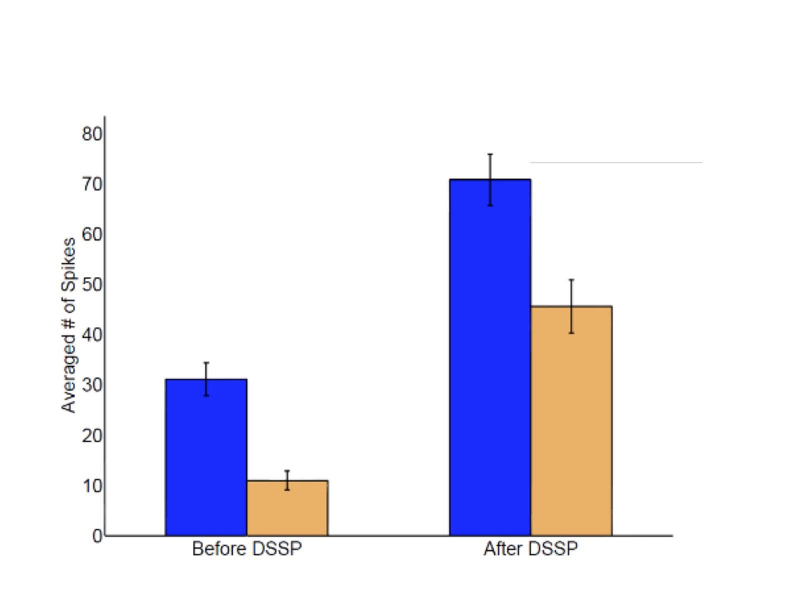
## Example: Localization results of an epileptic spike retrieved by DSSP

**Case 1** Sparse Bayes source reconstruction (Champagne) algorithm was used.




- With DSSP, localization was in plausible brain areas. The spike-like voxel time-course was recovered.
- Without DSSP, an activity was localized to obviously wrong location (outside of or near the skull).

## Summary of our study conducted to evaluate DSSP's performance for epilepsy MEG data




**Visually-identified spikes**

**Successfully-localized spikes**

**Data from 10 patients were analyzed with and without DSSP**

DSSP is a powerful tool to retrieve clinical information when large VNS artifacts exist.

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Cai, C., Xu, J., Knowlton, R., Sekihara, K., Nagarajan, S.S., Kirsch, H. Evaluation of a dual signal subspace projection algorithm in magnetoencephalographic recordings from patients with intractable epilepsy and vagus nerve stimulators. submitted to NeuroImage.

# Comparison between DSSP and tSSS algorithms

These algorithms differ only in the projectors used in Step#1

DSSP

Pseudo-signal subspace projector

tSSS

SSS internal subspace (pseudo) projector

Interference overlapped data

↓

Step#1

De-signaling by spatial-domain SSP

↓

Step#2

Estimation of time-domain interference subspace

↓

Step#3

Interference removal by time-domain SSP

↓

Interference removed data

# Difference in their performances

**Non-MEG applications**

tSSS cannot be applied to data taken from non-helmet-type sensor arrays.

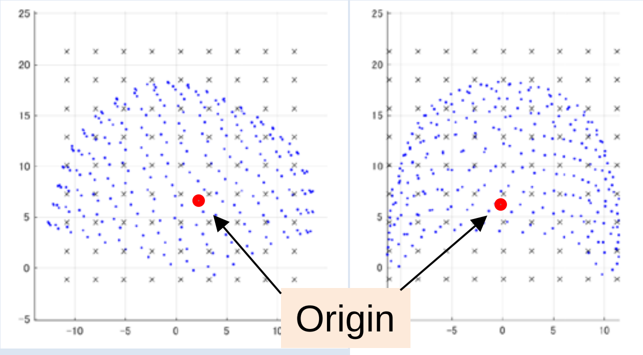
DSSP can be and has been used in artifact removal in functional spinal cord biomagnetic imaging that uses a flat sensor array.

**MEG applications (using helmet sensor arrays)**

Their overall performances are more or less the same, because  the performances of the de-signaling projectors are similar,.

There is one difference.
In tSSS, the de-signaling (internal) region can be smaller than the source space, and this fact requires users to choose the origin location with some care.

# tSSS and DSSP de-signaling (internal) regions: Example with CTF sensor array

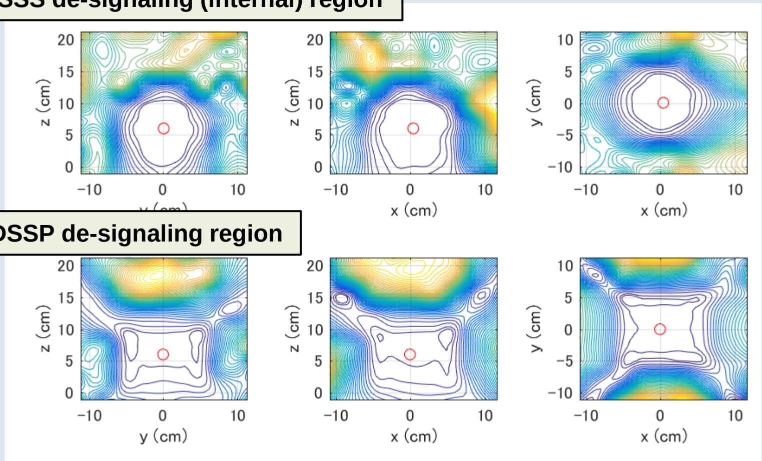



Plots of de-signaling gain $\phi = \frac{\left\|(\boldsymbol{I}-\boldsymbol{P})\boldsymbol{B}_S\right\|}{\left\|\boldsymbol{B}_S\right\|}$

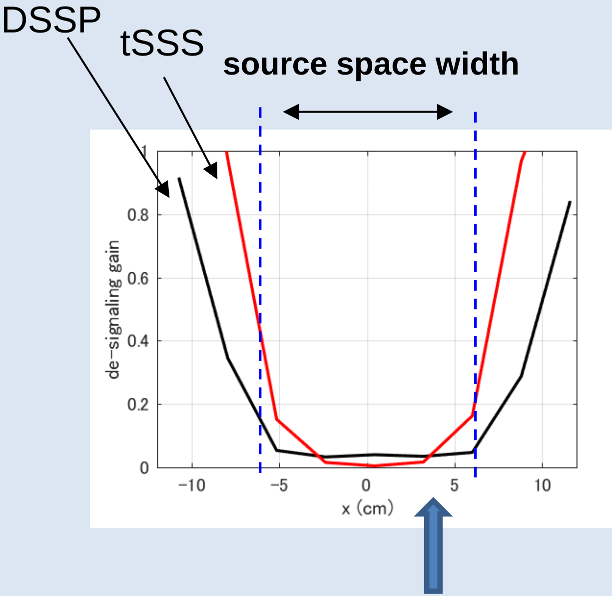
$\boldsymbol{P}$: internal subspace projector (tSSS)

: pseudo signal subspace projector (DSSP)

**tSSS de-signaling (internal) region**

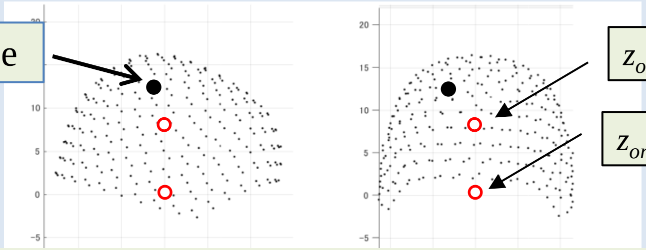



**DSSP de-signaling region**




**De-signaling region for tSSS is a bit smaller than the source space.**

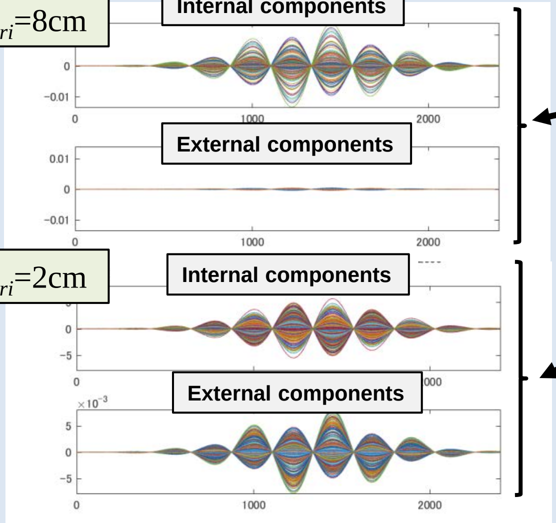
# Two cases where the origin is appropriately and inappropriately set

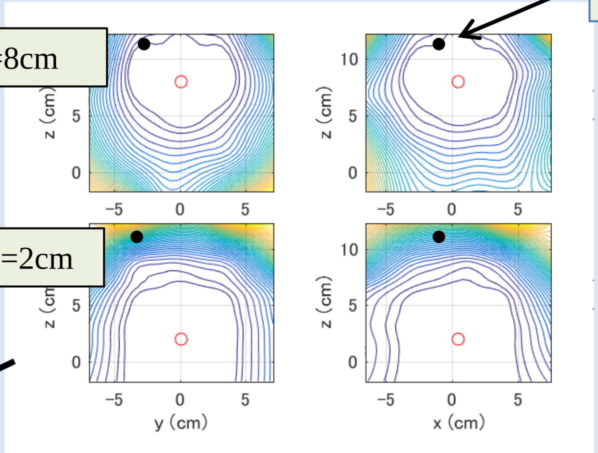



Origin location    Signal source of interest

Plot of de-signaling gain:

$$\phi = \frac{\left\| (\boldsymbol{I} - \boldsymbol{P}_{\text{int}}) \boldsymbol{B}_S \right\|}{\left\| \boldsymbol{B}_S \right\|}$$

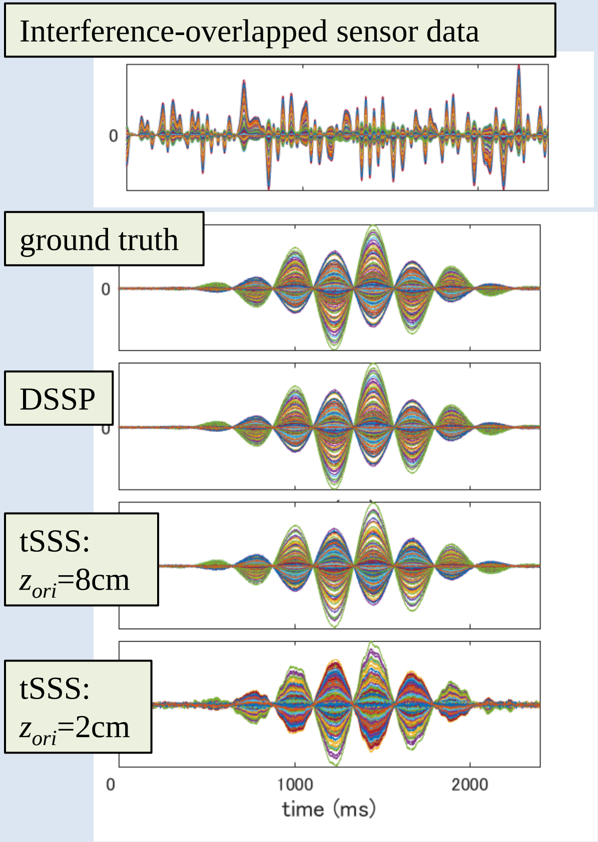


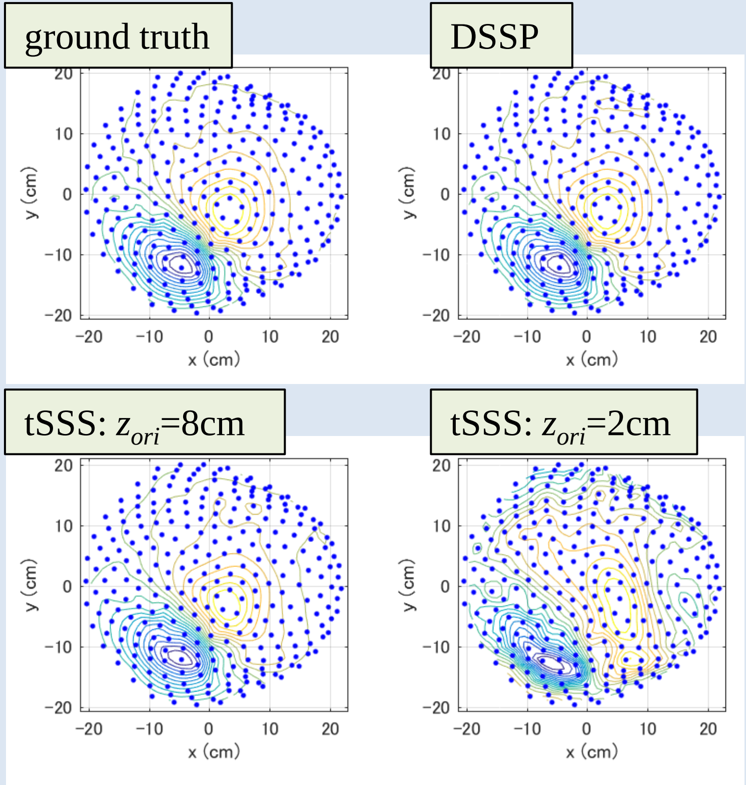




signal source location and
de-signaling region

**If signal source is outside the internal region, the signal magnetic field has external components.**

# What happens to the results of tSSS when signal magnetic field has external components




tSSS results are distorted, if signal magnetic field has external components, because the algorithm considers the signal external components to be a part of interference and time-domain SSP removes these components.

## Related presentation

The beamspace DSSP (bDSSP) algorithm is presented in Poster #M-084 in Poster Session M2, 6:00—7:30PM on Monday.

The bDSSP algorithm is an extended version of DSSP.

I can selectively detect a deep source by suppressing stronger signals from superficial sources.

# Related publications


## DSSP algorithm

Sekihara, K., Kawabata, Y., Ushio, S., Sumiya, S., Kawabata, S., Adachi, Y., & Nagarajan, S. S. (2016). Dual signal subspace projection (DSSP): a novel algorithm for removing large interference in biomagnetic measurements. *Journal of Neural Engineering*, *13*(3), 036007.

Sekihara K. & Nagarajan S.S. Dual Signal Subspace Projection: A powerful Algorithm for Interference Removal and Selective Detection of Signal Sources, Book Chapter contribution to “Magnetoencephalography: From Signals to Dynamic Cortical Networks, 2nd Edition”.

## Time-Domain Signal Subspace

Sekihara, K., & Nagarajan, S. S. (2017). Subspace-based interference removal methods for a multichannel biomagnetic sensor array. *Journal of Neural Engineering*, *14*(5), 051001.

## bDSSP algorithm

Sekihara, K., Adachi, Y., Kubota, H. K., Cai, C., & Nagarajan, S. S. (2018). Beamspace dual signal space projection (bDSSP): a method for selective detection of deep sources in MEG measurements. *Journal of neural engineering*, *15*(3), 036026.

## Application to VNS artifact removal

Cai, C., Xu, J., Knowlton, R., Sekihara, K., Nagarajan, S.S., Kirsch, H. Evaluation of a dual signal subspace projection algorithm in magnetoencephalographic recordings from patients with intractable epilepsy and vagus nerve stimulators. submitted to NeuroImage.


**QR codes for the links to download the reprints are available at Poster #M-084 in Poster Session M2, 6:00—7:30PM on Monday.**


Thank you very much for your attention.